Python for Data Science

ACTL3143 & ACTL5111 Deep Learning for Actuaries

Patrick Laub

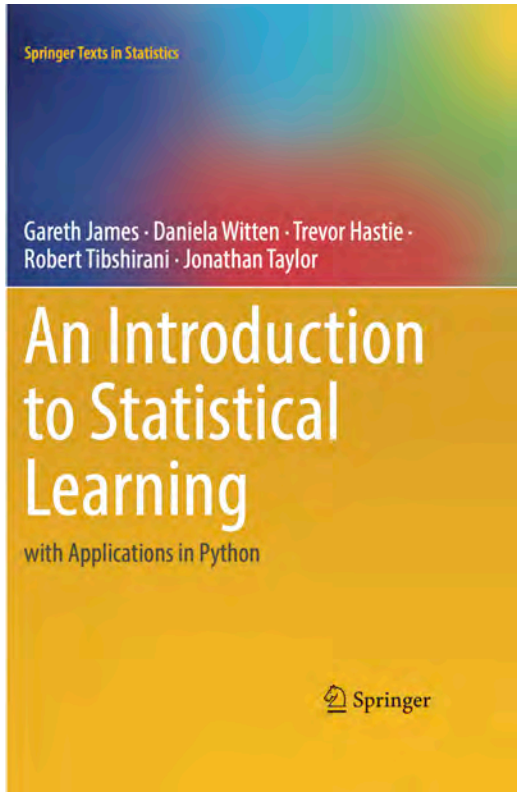
Preliminary notes

Readings for this topic:

Week	Readings (ACTL3142 Revision)
1	James et al. (2021): Chapter 2, Sections 3.1, 3.2, and 5.1.1
2	James et al. (2021): Section 3.3.1, 4.1, 4.2, 4.3

For ACTL3143 students, these slides are mostly a revision of ACTL3142 concepts but using Python instead of R.

For ACTL5111 students, if you have not taken ACTL5110, you need to self-study any surprising concepts in these slides ASAP.

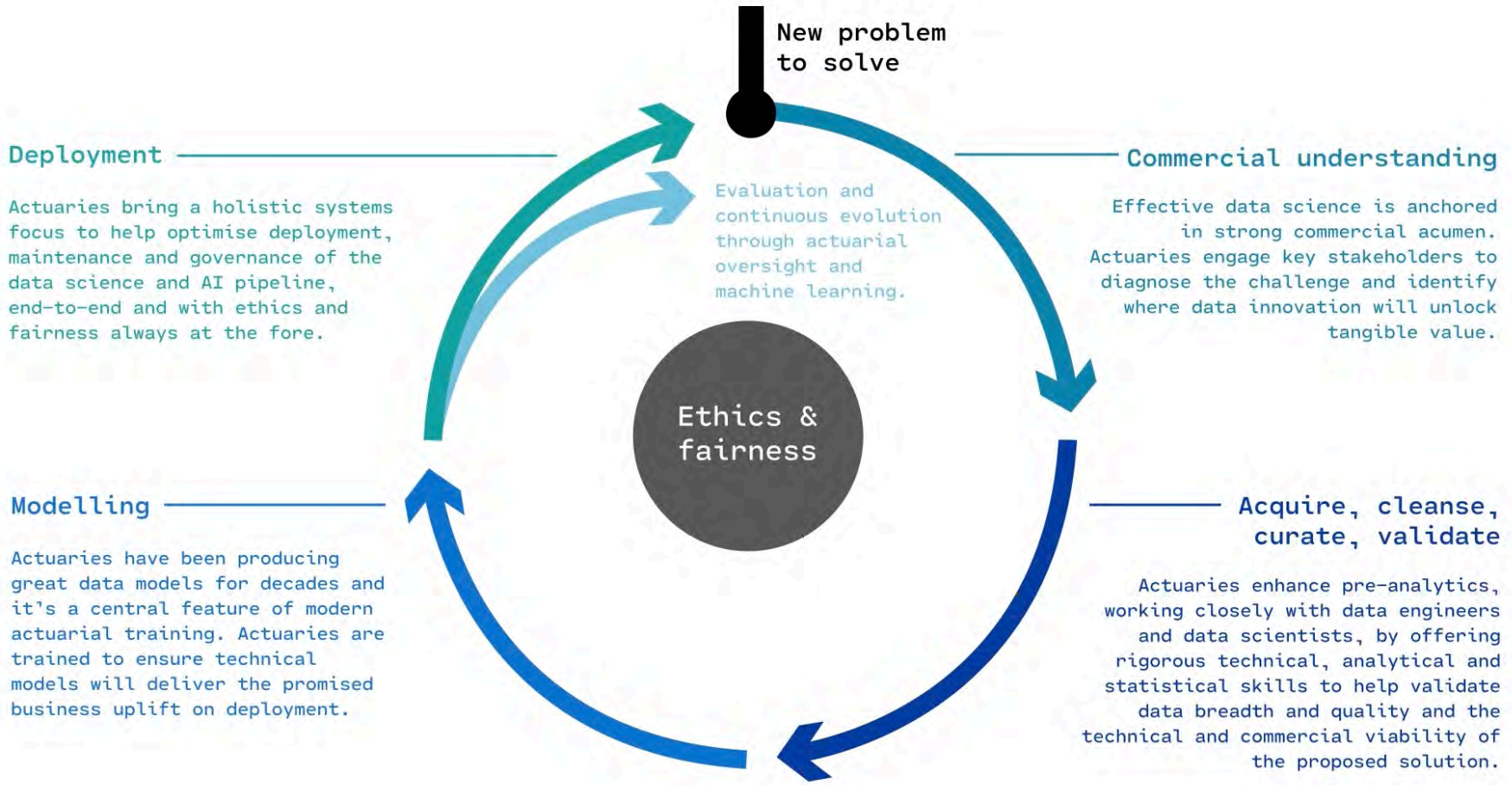


Introduction to Statistical Learning with ~~R~~ Python

Lecture Outline

- **Machine learning workflow**
- Data science packages
- Linear Regression Recap
- Generalised Linear Models Recap
- California House Price Prediction (Setup)
- California House Price Prediction (Regression)

An entire ML project



ML life cycle

Source: Actuaries Institute.

Questions to answer in ML project

You fit a few models to the training set, then ask:

1. **(Selection)** Which of these models is the best?
2. **(Future Performance)** How good should we expect the final model to be on unseen data?

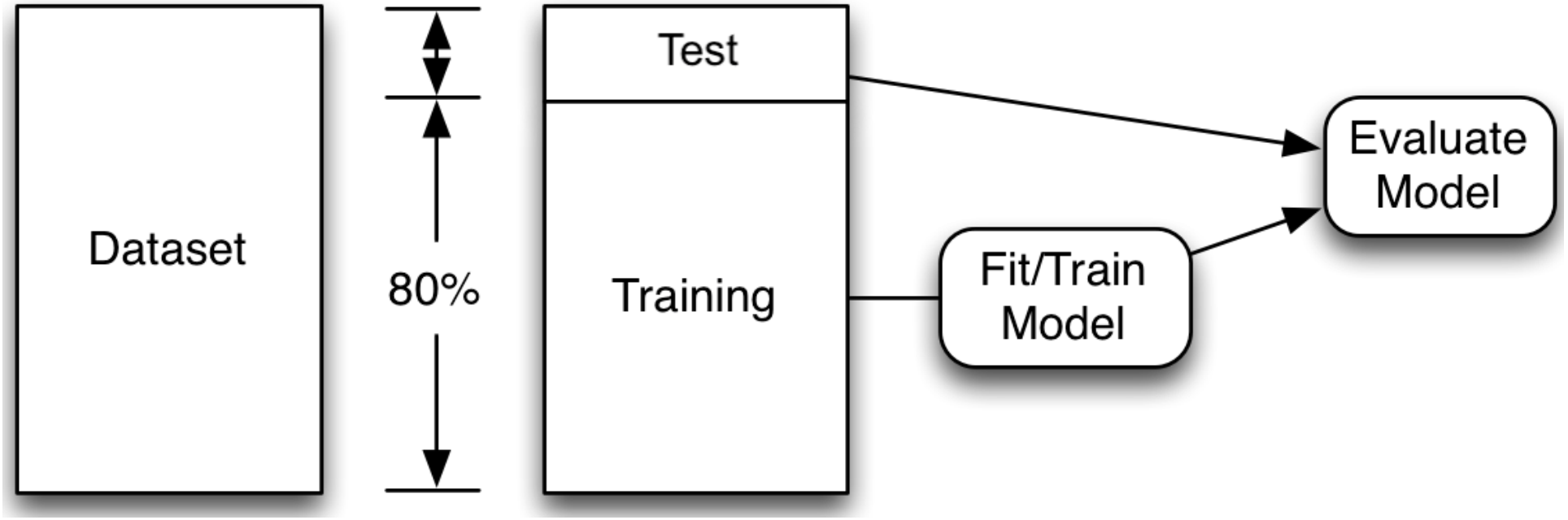
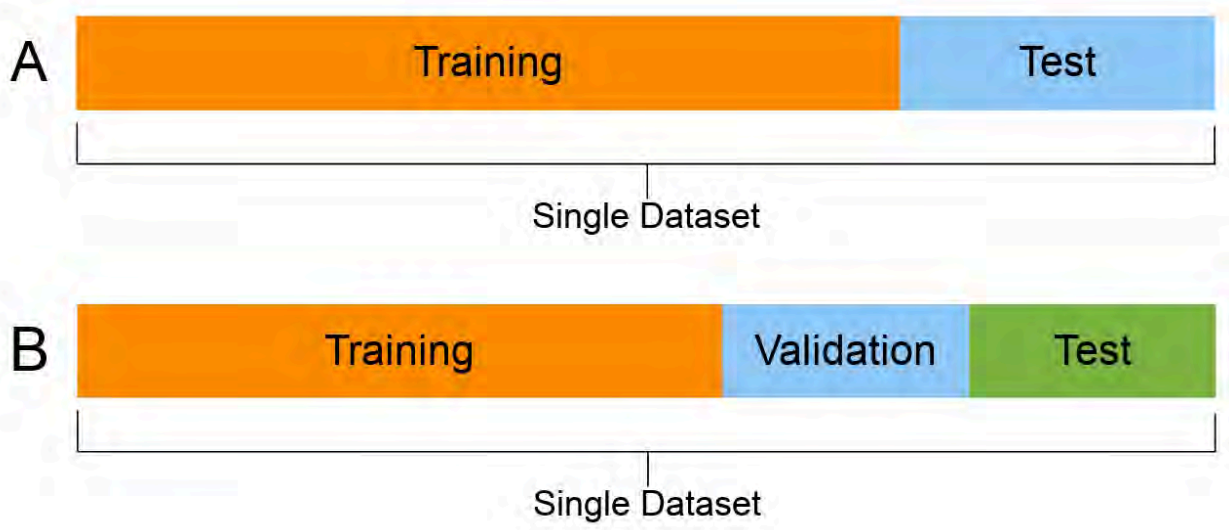


Illustration of a typical training/test split.

Adapted from: Heaton (2022), Applications of Deep Learning.

Validation set approach



Splitting the data.

1. For each model, fit it to the *training set*.
2. Compute the error for each model on the *validation set*.
3. Select the model with the lowest validation error.
4. Compute the error of the final model on the *test set*.

Read Section 5.1.1 of James et al. (2021) for details.

Source: Wikipedia.

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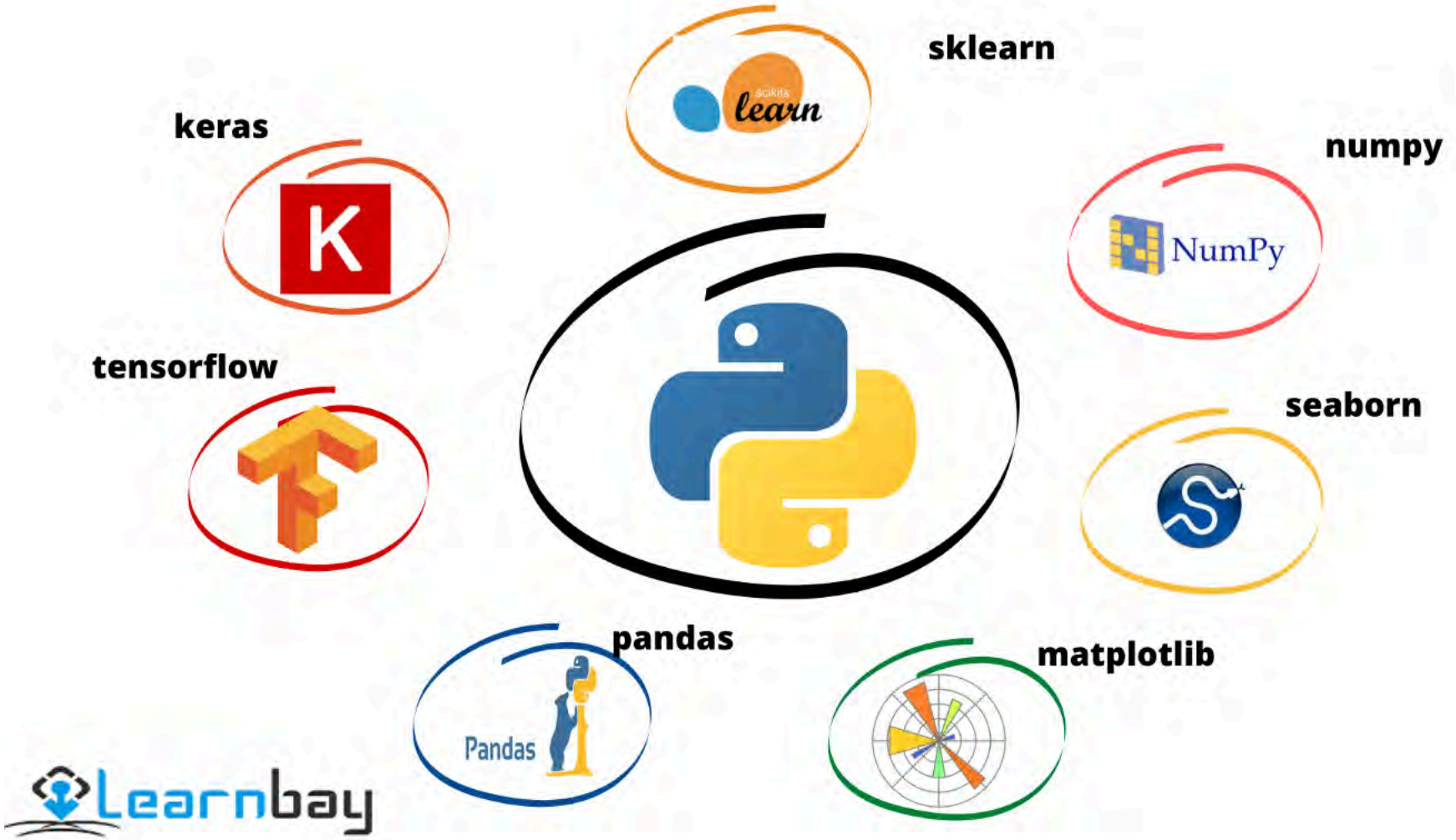
Data science packages



Common data science packages

Source: Learnbay.co, Python libraries for data analysis and modeling in Data science, Medium.

Imports for these slides

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from sklearn.datasets import fetch_california_housing
from sklearn.linear_model import LinearRegression, LogisticRegression, PoissonRegressor
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.tree import DecisionTreeClassifier, plot_tree
```

Numpy for matrices

Numpy is Python's standard approach for any low-level numerical mathematics tasks (matrix processing, linear algebra)

```
1  a = np.array([1, 2, 3, 4])
2  a.mean(), a.std()
```

```
(np.float64(2.5),
np.float64(1.118033988749895))
```

```
1  a * 2 + 1 # vectorised arithmetic
```

```
array([3, 5, 7, 9])
```

```
1  M = np.arange(6).reshape(2, 3)
2  M
```

```
array([[0, 1, 2],
       [3, 4, 5]])
```

```
1  M.shape, M.sum(axis=0)
```

```
((2, 3), array([3, 5, 7]))
```

Travis Oliphant, creator of Numpy (and Scipy). Now >2K contributors, ~450K LOC (C), 19B installs (Open Hub and ClickPy)

Also has random number generation functionality

```
1  rng = np.random.default_rng(0)
2  x = rng.normal(size=200)
3  y = 2 * x + rng.normal(size=200)
```

Pandas for DataFrames

Used for tabular data: loading, cleaning, processing, saving

```
df = pd.DataFrame({
    "age": [25, 32, 41, 28, 36],
    "salary": [45_000, 65_000, 80_000, 55
})
df.head()
```

	age	salary
0	25	45000
1	32	65000
2	41	80000
3	28	55000
4	36	72000

Wes McKinney, creator of Pandas. Now >4K contributors, ~475K LOC (Python), 14B installs (Open Hub and ClickPy)

```
df.shape          # (rows, cols)
df.columns.tolist()
```

```
['age', 'salary']
```

```
df["age"]         # column
df.iloc[0]        # first row
```

```
age           25
salary     45000
Name: 0, dtype: int64
```

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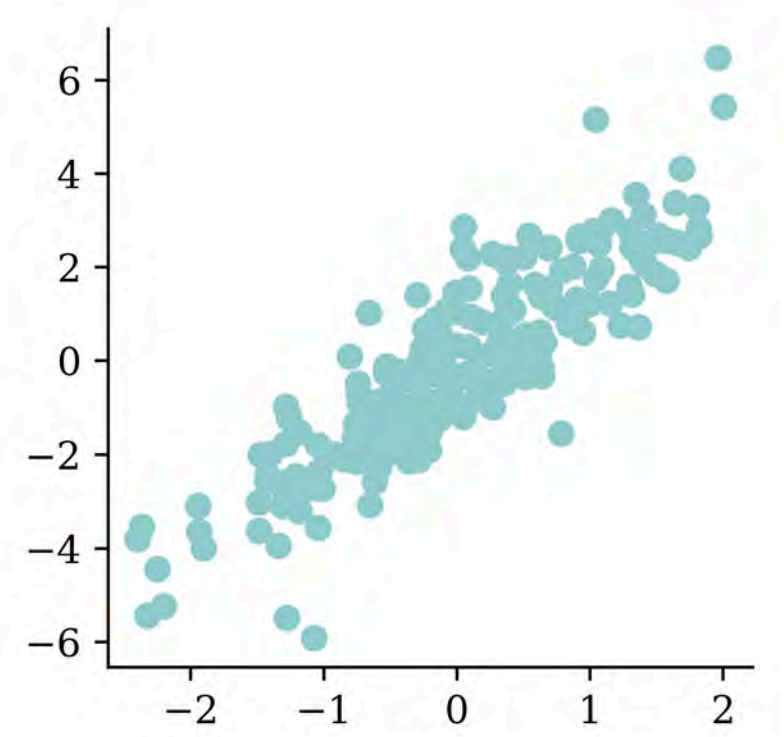
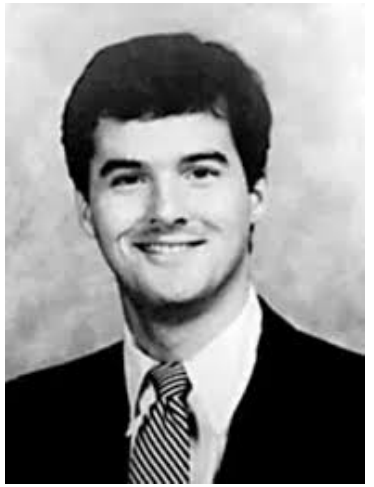
Matplotlib for plots

The traditional way to generate plots; similar (in spirit) to R's base `plot()`

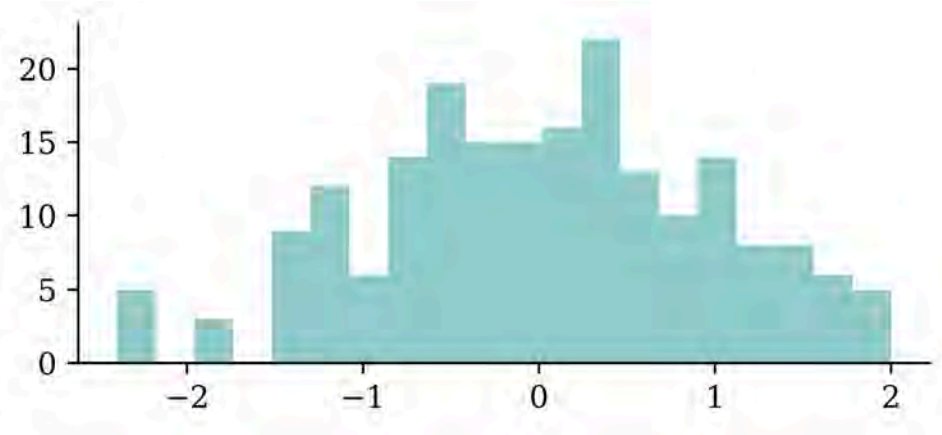
```
1 plt.figure(figsize=(3, 3))
2 plt.scatter(x, y);
```





John D. Hunter (1968–2012), creator of Matplotlib. Now has ~290K LOC (Python), >2K contributors, 4B installs (Open Hub and ClickPy)

```
1 plt.hist(x, bins=20);
```



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Scikit-learn for machine learning

Every sklearn estimator has the same three methods:

- `fit(X, y)`: learn the parameters of the model
- `predict(X)`: apply the model
- `score(X, y)`: apply the model and calculate a metric

```python
X = df[["age"]] # [[ ]] gives a 2D DataFrame
y = df["salary"]
model = LinearRegression()
model.fit(X, y)
y_pred = model.predict(X)
print(f"R^2 = {model.score(X, y):.3f}")
```

```
R^2 = 0.977
```

Set aside a fraction for a test set

Off-screen I have loaded up a larger `X` & `y` dataset:

```
print(X.shape, y.shape)
```

```
(10000, 4) (10000,)
```

We can split into train and test sets (in a random but reproducible way):

```
X_train, X_test, y_train, y_test = train_test_split(X, y, random_state=42)
print(X_train.shape, X_test.shape, y_train.shape, y_test.shape)
```

```
(7500, 4) (2500, 4) (7500,) (2500,)
```

Note: Compare `X_`/`y_` names, capitals & lowercase.

Split three ways:

```
X_main, X_test, y_main, y_test = train_test_split(X, y, test_size=0.2, random_state=1)

# As 0.25 x 0.8 = 0.2
X_train, X_val, y_train, y_val = train_test_split(X_main, y_main, test_size=0.25, random_

X_train.shape, X_val.shape, X_test.shape
```

```
((6000, 4), (2000, 4), (2000, 4))
```

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Single Linear Regression (SLR)

Scenario: use *head circumference (mm)* to predict *age (weeks)* of a fetal baby

$$\boldsymbol{x} = \begin{pmatrix} 105 \\ 120 \\ 110 \\ 117 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 20 \\ 24 \\ 22 \\ 23 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 20.3 \\ 24.0 \\ 21.5 \\ 23.2 \end{pmatrix}$$

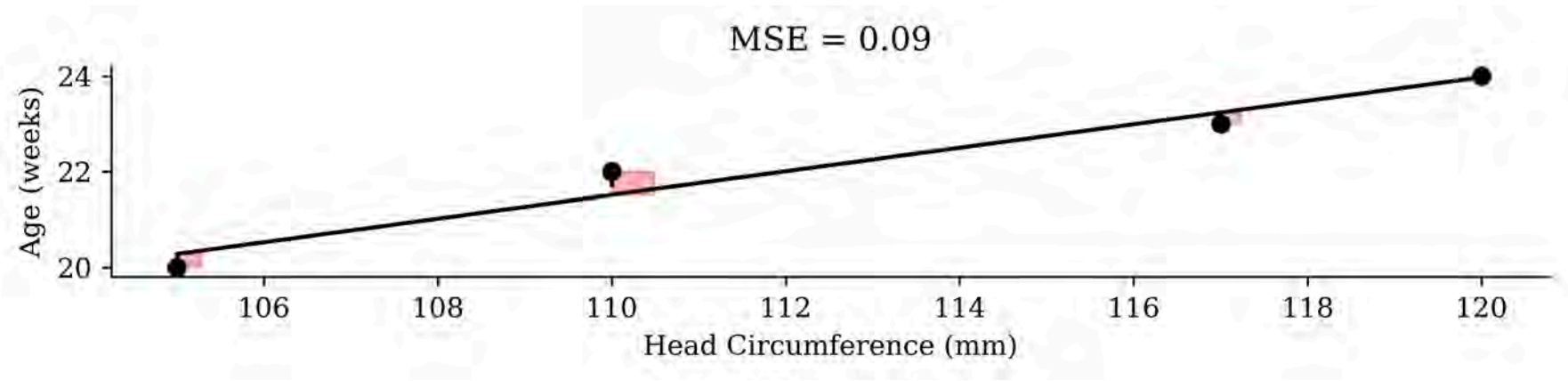
$$\hat{y}(x) = \beta_0 + \beta_1 x$$

```
babies = pd.DataFrame({
    "circumference": [105, 120, 110, 117],
    "age": [20, 24, 22, 23],
})
X = babies[["circumference"]]
y = babies["age"]
model = LinearRegression()
model.fit(X, y)
model.predict(X)
```

```
array([20.28, 23.97, 21.51, 23.24])
```

Mean squared error

$$\boldsymbol{y} = \begin{pmatrix} 20 \\ 24 \\ 22 \\ 23 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 20.3 \\ 24.0 \\ 21.5 \\ 23.2 \end{pmatrix}, \quad \boldsymbol{y} - \hat{\boldsymbol{y}} = \begin{pmatrix} -0.3 \\ 0 \\ 0.5 \\ -0.2 \end{pmatrix}.$$



$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \frac{1}{4}\left[(-0.3)^2 + 0^2 + 0.5^2 + (-0.2)^2\right] = 0.09.$$

$$\text{RSS} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = 0.38.$$

MSE in sklearn

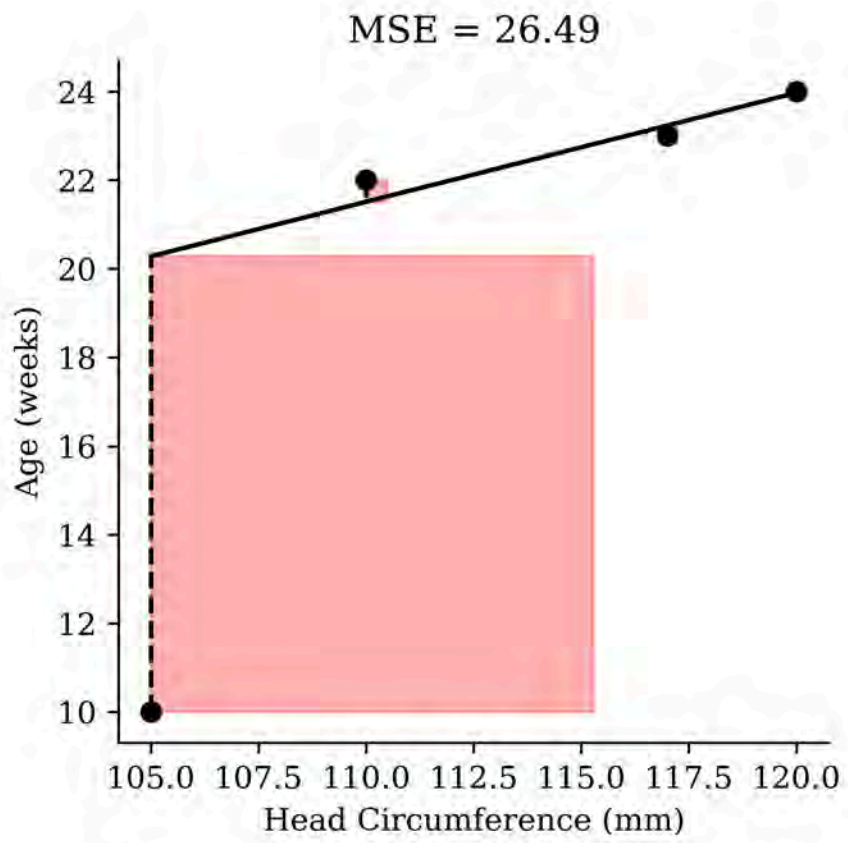
Rather than computing the formula by hand, use `mean_squared_error` from `sklearn.metrics`.

```
1 mean_squared_error(y, model.predict(X))
```

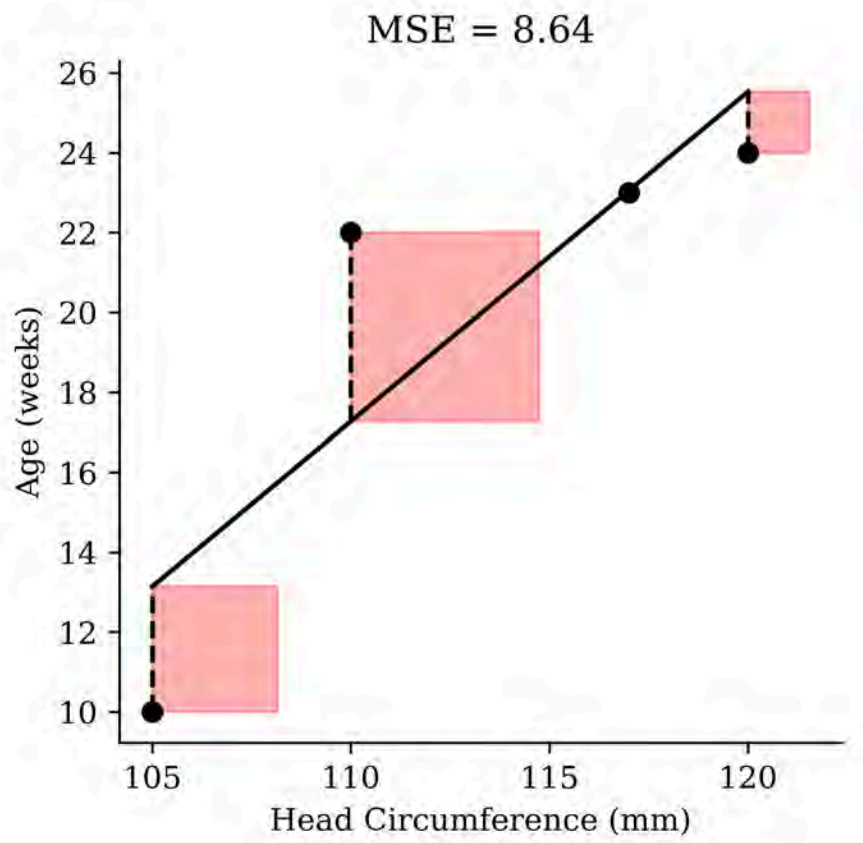
```
0.0932971014492754
```

Sensitive to outliers

If we instead had $\boldsymbol{y} = \begin{pmatrix} 10 & 24 & 22 & 23 \end{pmatrix}^\top$.



The previous model



New model after training

Multiple Linear Regression (MLR)

Scenario: use *temperature (°C)* **and** *humidity (%)* to predict *rainfall (mm)*

$$\boldsymbol{X} = \begin{pmatrix} 27 & 80 \\ 32 & 70 \\ 28 & 72 \\ 22 & 86 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 6 \\ 7 \\ 6 \\ 4 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 5.7 \\ 7.2 \\ 5.9 \\ 4.2 \end{pmatrix}$$

$$\hat{y}(\boldsymbol{x}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p = \beta_0 + \langle \boldsymbol{\beta}, \boldsymbol{x} \rangle$$

```python
df = pd.DataFrame({
    "temperature": [27, 32, 28, 22],
    "humidity": [80, 70, 72, 86],
    "rainfall": [6, 7, 6, 4]
})
X = df[["temperature", "humidity"]]
y = df["rainfall"]

model = LinearRegression()
model.fit(X, y)
model.predict(X)
```

```
array([5.74, 7.2 , 5.88, 4.18])
```

Predict by hand

The intercept β_0 is in `model.intercept_` and the slopes $\beta_1, \ldots, \beta_p$ are in `model.coef_`.

`model.predict(X)` gives us:

```
1 model.predict(X)
```

```
array([5.74, 7.2 , 5.88, 4.18])
```

We can reproduce the first entry with a `for` loop:

```
1 prediction = model.intercept_
2 for beta_j, x_0j in zip(model.coef_, X.iloc[0]):
3     prediction += beta_j * x_0j
4 prediction
```

```
np.float64(5.739526411657559)
```

MLR's design matrix

Scenario: use *temperature (°C)* **and** *humidity (%)* to predict *rainfall (mm)*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 27 & 80 \\ 1 & 32 & 70 \\ 1 & 28 & 72 \\ 1 & 22 & 86 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 6 \\ 7 \\ 6 \\ 4 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 5.7 \\ 7.2 \\ 5.9 \\ 4.2 \end{pmatrix}$$

$$\hat{y}(\boldsymbol{x}) = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p = \langle \boldsymbol{\beta}, \boldsymbol{x} \rangle$$

```
1 print(X.to_numpy())
```

```
[[27 80]
 [32 70]
 [28 72]
 [22 86]]
```

```
1 X_des = np.column_stack([np.ones(len(X)), X.to_numpy()])
2 print(X_des)
```

```
[[ 1. 27. 80.]
 [ 1. 32. 70.]
 [ 1. 28. 72.]
 [ 1. 22. 86.]]
```

MLR is just matrix equations

$$\boldsymbol{\beta} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top \boldsymbol{y}$$

```
1 model = LinearRegression()
2 model.fit(X, y)
3 print(model.intercept_, model.coef_)
```

```
-5.5100182149362436 [0.34 0.03]
```

```
1 beta = np.linalg.solve(
2     X_des.T @ X_des, X_des.T @ y)
3 beta
```

```
array([-5.51,  0.34,  0.03])
```

$$\hat{\boldsymbol{y}} = \boldsymbol{X}\boldsymbol{\beta}.$$

```
1 model.predict(X)
```

```
array([5.74, 7.2 , 5.88, 4.18])
```

```
1 X_des @ beta
```

```
array([5.74, 7.2 , 5.88, 4.18])
```

Note, you should just use the left column. The right column's version is for pedagogical purposes (I'll do this kind of thing again in the upcoming slides).

MLR after normalising the inputs

Scenario: use *temperature (z-score)* **and** *humidity (z-score)* to predict *rainfall (mm)*

$$\boldsymbol{X} = \begin{pmatrix} 1 & -0.06 & 0.41 \\ 1 & 1.15 & -0.95 \\ 1 & 0.18 & -0.68 \\ 1 & -1.28 & 1.22 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 6 \\ 7 \\ 6 \\ 4 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 5.7 \\ 7.2 \\ 5.9 \\ 4.2 \end{pmatrix}$$

$$\hat{y}(\boldsymbol{x}) = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p = \langle \boldsymbol{\beta}, \boldsymbol{x} \rangle$$

```
scaler = StandardScaler()
scaler.fit(X)
X_sc = pd.DataFrame(scaler.transform(X), columns=X.columns)

model_norm = LinearRegression()
model_norm.fit(X_sc, y)
print(model_norm.intercept_, model_norm.coef_, model_norm.predict(X_sc))
```

```
5.75 [1.22 0.16] [5.74 7.2  5.88 4.18]
```

```
print(model.intercept_, model.coef_, model.predict(X))
```

```
-5.5100182149362436 [0.34 0.03] [5.74 7.2  5.88 4.18]
```

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SLR with categorical inputs

Scenario: use *property type* to predict its *sale price ($)*

$$\boldsymbol{x} = \begin{pmatrix} \text{Unit} \\ \text{TownHouse} \\ \text{TownHouse} \\ \text{House} \end{pmatrix} \longrightarrow \boldsymbol{X} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 300000 \\ 500000 \\ 510000 \\ 700000 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 300000 \\ 505000 \\ 505000 \\ 700000 \end{pmatrix}$$

$$\hat{y}(x) = \beta_0 + \beta_1 \underbrace{I(x = \text{Unit})}_{=:x_1} + \beta_2 \underbrace{I(x = \text{TownHouse})}_{=:x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

```python
df = pd.DataFrame({
    "type": pd.Categorical(["Unit", "TownHouse", "TownHouse", "House"]),
    "price": [300000, 500000, 510000, 700000],
})
X = pd.get_dummies(df["type"], drop_first=True, dtype=int)
y = df["price"]
model = LinearRegression()
model.fit(X, y)
print(model.intercept_, model.coef_, model.predict(X))
```

```
700000.0 [-195000. -400000.] [300000. 505000. 505000. 700000.]
```

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Changing the base case

Scenario: use *property type* to predict its *sale price ($)*

$$\boldsymbol{x} = \begin{pmatrix} \text{Unit} \\ \text{TownHouse} \\ \text{TownHouse} \\ \text{House} \end{pmatrix} \longrightarrow \boldsymbol{X} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 300000 \\ 500000 \\ 510000 \\ 700000 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 300000 \\ 505000 \\ 505000 \\ 700000 \end{pmatrix}$$

$$\hat{y}(x) = \beta_0 + \beta_1 \underbrace{I(x = \text{TownHouse})}_{=:x_1} + \beta_2 \underbrace{I(x = \text{House})}_{=:x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

```
1 df["type"] = df["type"].cat.reorder_categories(["Unit", "TownHouse", "House"])
2 X = pd.get_dummies(df["type"], drop_first=True, dtype=int)
3
4 model = LinearRegression()
5 model.fit(X, y)
6 print(model.intercept_, model.coef_, model.predict(X))
```

```
299999.99999999994 [205000. 400000.] [300000. 505000. 505000. 700000.]
```

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Changing the target's units

Scenario: use *property type* to predict its *sale price ($ooo,ooo's)*

$$\boldsymbol{x} = \begin{pmatrix} \text{Unit} \\ \text{TownHouse} \\ \text{TownHouse} \\ \text{House} \end{pmatrix} \longrightarrow \boldsymbol{X} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 0.3 \\ 0.5 \\ 0.51 \\ 0.7 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} 0.3 \\ 0.505 \\ 0.505 \\ 0.7 \end{pmatrix}$$

$$\hat{y}(x) = \beta_0 + \beta_1 \underbrace{I(x = \text{TownHouse})}_{=:x_1} + \beta_2 \underbrace{I(x = \text{House})}_{=:x_2} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

```
df["price"] /= 1e6
y = df["price"]

model = LinearRegression()
model.fit(X, y)
print(model.intercept_, model.coef_, model.predict(X))
```

```
0.2999999999999999 [0.21 0.4 ] [0.3  0.51 0.51 0.7 ]
```

Dates as categorical inputs

Imagine you had dataset with 157,209 rows and one covariate column was ‘date of birth’. There were 27,485 unique values for the date of birth column. You make a linear regression with the date of birth as an input, and treat it as a categorical variable.

Pandas/sklearn would make a design matrix of size $157,209 \times 27,485$. This has 4,320,889,365 numbers in it. Each number is stored as 8 bytes, so the matrix is roughly

$$34,000,000,000 \text{ bytes} = 34,000,000 \text{ KB} = 34,000 \text{ MB} = 34 \text{ GB}.$$

Question: How do we avoid this situation of crashing our computer with an ‘out of memory’ error?

Solution: Don’t train on the date of birth — at least, not as a categorical variable. Turn it into a numerical variable, e.g. age. If the date column were not ‘date of birth’, maybe calculate the number of days since some reference date, or split into day, month, year columns.

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MLR with categorical targets

Scenario: use *tumour radius (mm)* and *ages (years)* to predict *benign* **or** *malignant*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 14.8 & 48 \\ 1 & 16.4 & 49 \\ 1 & 15.5 & 47 \\ 1 & 14.2 & 46 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} \text{Benign} \\ \text{Malignant} \\ \text{Benign} \\ \text{Malignant} \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix}$$

Step 1: Make a linear prediction

$$\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p, \quad \text{a.k.a.} \quad \langle \boldsymbol{\beta}, \boldsymbol{x} \rangle.$$

Step 2: Shove it through some function to make it a probability

$$\sigma(\langle \boldsymbol{\beta}, \boldsymbol{x} \rangle)$$

Interpret that as being the probability of the positive class.

Step 3: Make a decision based on that probability. E.g., if the probability is greater than 0.5, predict the positive class.

Binomial regression with sklearn

Scenario: use *tumour radius (mm)* and *ages (years)* to predict *benign* **or** *malignant*

$$\boldsymbol{y} = \begin{pmatrix} \text{Benign} \\ \textbf{Malignant} \\ \text{Benign} \\ \textbf{Malignant} \end{pmatrix}, \quad \sigma(\boldsymbol{X}\boldsymbol{\beta}) = \begin{pmatrix} 39\% \\ 55\% \\ 59\% \\ 48\% \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} \text{Benign} \\ \text{Malignant} \\ \text{Malignant} \\ \text{Benign} \end{pmatrix}$$

Load the data, fit a GLM, make predictions:

```
patients = pd.DataFrame({
    "radius": [14.8, 16.4, 15.5, 14.2],
    "age": [48, 49, 47, 46],
    "tumour": ["Benign", "Malignant", "Benign", "Malignant"],
})
X = patients[["radius", "age"]]
y = patients["tumour"]

model = LogisticRegression(C=np.inf)
model.fit(X, y)
model.predict(X)
```

```
array(['Benign', 'Malignant', 'Malignant', 'Benign'], dtype=object)
```

Binomial regression I

Scenario: use *tumour radius (mm)* and *ages (years)* to predict *benign* **or** *malignant*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 14.8 & 48 \\ 1 & 16.4 & 49 \\ 1 & 15.5 & 47 \\ 1 & 14.2 & 46 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} \text{Benign} \\ \textbf{Malignant} \\ \text{Benign} \\ \textbf{Malignant} \end{pmatrix}, \quad \boldsymbol{X\beta} = \begin{pmatrix} -0.46 \\ 0.18 \\ 0.37 \\ -0.09 \end{pmatrix}$$

```
model = LogisticRegression(C=np.inf)
model.fit(X, y);
```

Step 1: Make a linear prediction

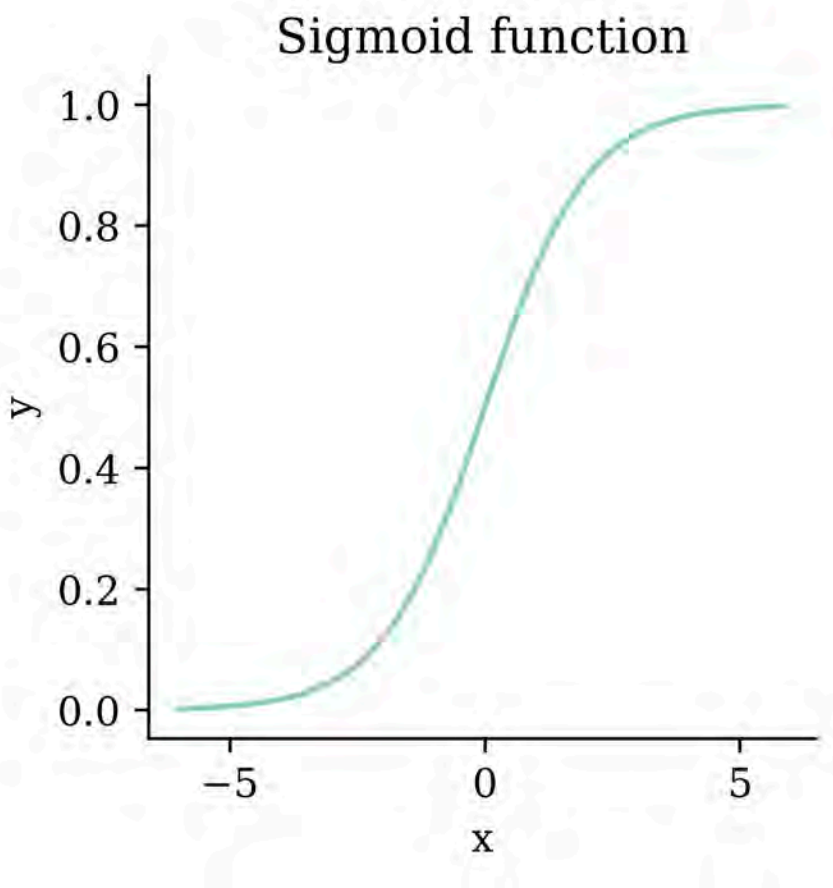
```
linear = model.decision_function(X)
linear
```

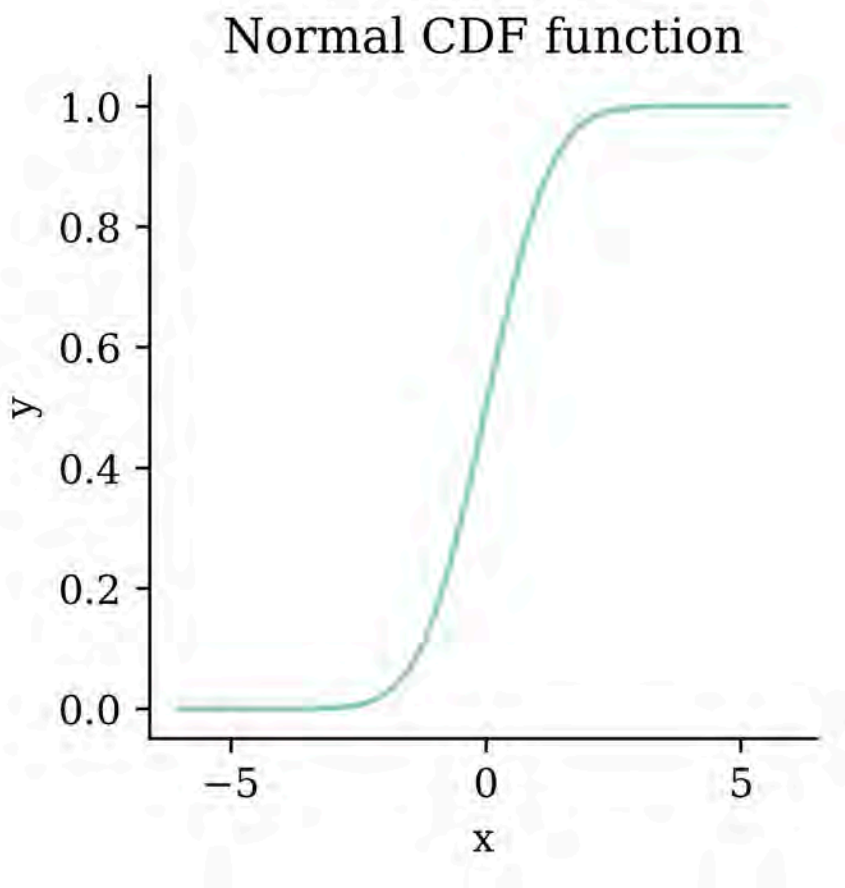
```
array([-0.46,  0.18,  0.37, -0.09])
```

```
X_des = np.column_stack([np.ones(len(X)), X.to_numpy()])
beta = np.concatenate([model.intercept_, model.coef_.flatten()])
X_des @ beta
```

```
array([-0.46,  0.18,  0.37, -0.09])
```

Logistic regression and probit regression





This give you **logistic regression**.

This gives you **probit regression**.

Logistic Regression

Scenario: use *tumour radius (mm)* and *ages (years)* to predict *benign* **or** *malignant*

$$\boldsymbol{y} = \begin{pmatrix} \text{Benign} \\ \textbf{Malignant} \\ \text{Benign} \\ \textbf{Malignant} \end{pmatrix}, \quad \boldsymbol{X\beta} = \begin{pmatrix} -0.46 \\ 0.18 \\ 0.37 \\ -0.09 \end{pmatrix}, \quad \sigma(\boldsymbol{X\beta}) = \begin{pmatrix} 39\% \\ 55\% \\ 59\% \\ 48\% \end{pmatrix}$$

```
model = LogisticRegression(C=np.inf)
model.fit(X, y)
linear = model.decision_function(X)
```

Step 2: Shove it through some function to make it a probability

```
sigmoid(linear)
```

```
array([0.39, 0.55, 0.59, 0.48])
```

```
model.predict_proba(X)[:, 1] # Probability of the positive class (Malignant)
```

```
array([0.39, 0.55, 0.59, 0.48])
```

Predicting categories

Scenario: use *tumour radius (mm)* and *ages (years)* to predict *benign* **or** *malignant*

$$\boldsymbol{y} = \begin{pmatrix} \text{Benign} \\ \textbf{Malignant} \\ \text{Benign} \\ \textbf{Malignant} \end{pmatrix}, \quad \sigma(\boldsymbol{X}\boldsymbol{\beta}) = \begin{pmatrix} 39\% \\ 55\% \\ 59\% \\ 48\% \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} \text{Benign} \\ \text{Malignant} \\ \text{Malignant} \\ \text{Benign} \end{pmatrix}$$

```
model = LogisticRegression(C=np.inf)
model.fit(X, y);
```

```
predict_probs = model.predict_proba(X)[:, 1]
cutoff = 0.5
predicted_tumours = np.where(predict_probs > cutoff, "Malignant", "Benign")
print(predicted_tumours)
```

```
['Benign' 'Malignant' 'Malignant' 'Benign']
```

Assessing accuracy in classification problems

Scenario: use *tumour radius (mm)* and *ages (years)* to predict *benign* **or** *malignant*

$$\boldsymbol{y} = \begin{pmatrix} \text{Benign} \\ \text{Malignant} \\ \text{Benign} \\ \text{Malignant} \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} \text{Benign} \\ \text{Malignant} \\ \text{Malignant} \\ \text{Benign} \end{pmatrix}$$

Accuracy:

$$\frac{1}{n}\sum_{i=1}^{n} I(y_i = \hat{y}_i)$$

Error rate:

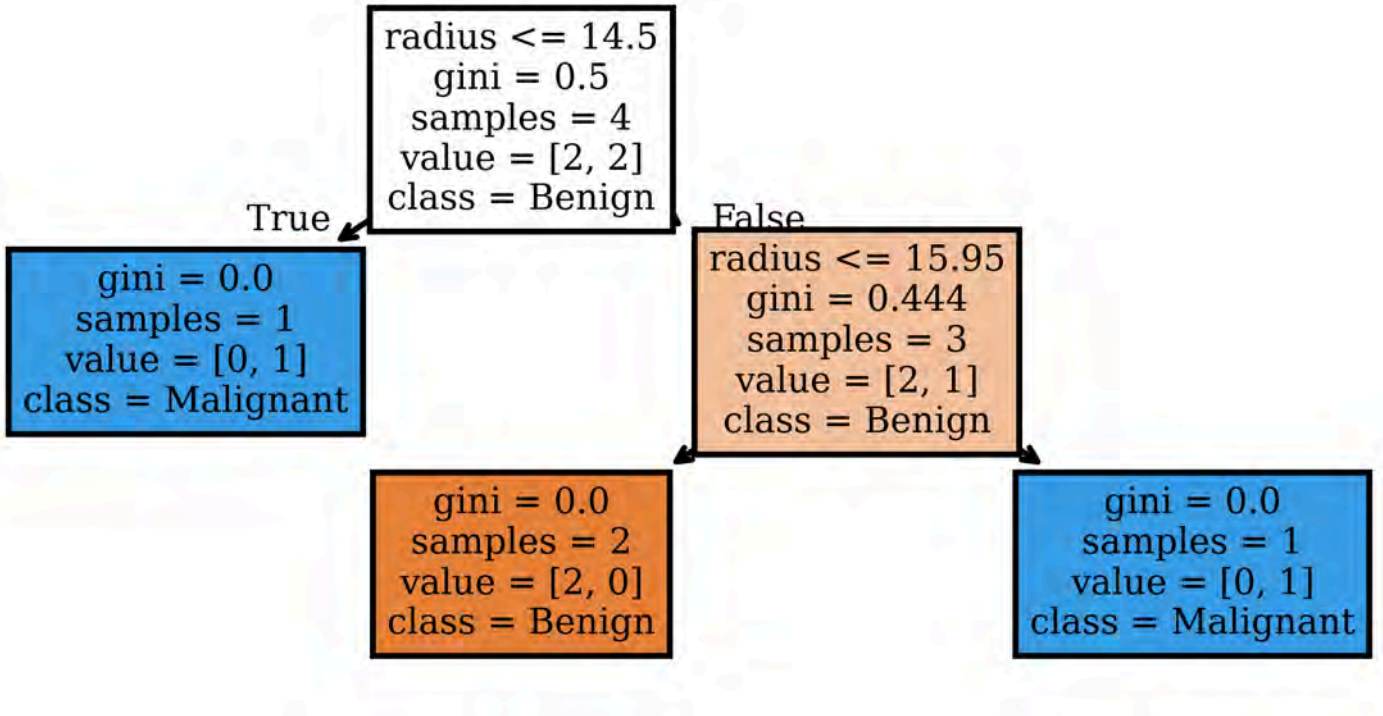
$$\frac{1}{n}\sum_{i=1}^{n} I(y_i \neq \hat{y}_i)$$

Note

The word “accuracy” is used often used loosely when discussing models, particularly regression models. In the context of classification it has this specific meaning.

Decision Tree

```
1 model = DecisionTreeClassifier(min_samples_split=2)
2 model.fit(X, y)
3
4 plot_tree(model, feature_names=list(X.columns), class_names=model.classes_, filled=True);
```



Poisson regression

Scenario: use *population density* to predict the daily **count of** *gun violence incidents*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 12 \\ 1 & 18 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 155 \\ 208 \\ 116 \\ 301 \end{pmatrix}, \quad \hat{\boldsymbol{y}} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \end{pmatrix}$$

Step 1: Make a linear prediction

$$\beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p, \quad \text{a.k.a.} \quad \langle \boldsymbol{\beta}, \boldsymbol{x} \rangle.$$

Step 2: Shove it through some function to make it positive

$$\exp(\langle \boldsymbol{\beta}, \boldsymbol{x} \rangle)$$

Interpret that as being the expected count.

Predict the mean of a Poisson distribution I

Scenario: use *population density* to predict the daily **count of** *gun violence incidents*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 12 \\ 1 & 18 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 155 \\ 208 \\ 116 \\ 301 \end{pmatrix}, \quad \boldsymbol{X\beta} = \begin{pmatrix} 4.82 \\ 5.35 \\ 5.03 \\ 5.67 \end{pmatrix}, \quad \hat{\boldsymbol{\mu}} = \begin{pmatrix} 125.1 \\ 211.3 \\ 154.3 \\ 289.4 \end{pmatrix}$$

Load the data, fit the model, and make a prediction.

```
guns = pd.DataFrame({
    "density": [10, 15, 12, 18],
    "incidents": [155, 208, 116, 301],
})
X = guns[["density"]]
y = guns["incidents"]

model = PoissonRegressor(alpha=0)
model.fit(X, y)

print(model.predict(X))
```

```
[125.08 211.28 154.26 289.38]
```

Predict the mean of a Poisson distribution II

Scenario: use *population density* to predict the daily **count of** *gun violence incidents*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 12 \\ 1 & 18 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 155 \\ 208 \\ 116 \\ 301 \end{pmatrix}, \quad \boldsymbol{X\beta} = \begin{pmatrix} 4.82 \\ 5.35 \\ 5.03 \\ 5.67 \end{pmatrix}, \quad \hat{\boldsymbol{\mu}} = \begin{pmatrix} 125.1 \\ 211.3 \\ 154.3 \\ 289.4 \end{pmatrix}$$

These are the linear coefficients:

```
1 print(model.intercept_, model.coef_)
```

```
3.7803728824634275 [0.1]
```

Step 1: Make linear predictions

```
1 X_des = np.column_stack([np.ones(len(X)), X.to_numpy()])
2 beta = np.concatenate([[model.intercept_], model.coef_])
3 linear = X_des @ beta
4 print(linear)
```

```
[4.83 5.35 5.04 5.67]
```

Predict the mean of a Poisson distribution III

Scenario: use *population density* to predict the daily **count of** *gun violence incidents*

$$\boldsymbol{X} = \begin{pmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 12 \\ 1 & 18 \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} 155 \\ 208 \\ 116 \\ 301 \end{pmatrix}, \quad \boldsymbol{X\beta} = \begin{pmatrix} 4.82 \\ 5.35 \\ 5.03 \\ 5.67 \end{pmatrix}, \quad \hat{\boldsymbol{\mu}} = \begin{pmatrix} 125.1 \\ 211.3 \\ 154.3 \\ 289.4 \end{pmatrix}$$

Step 2: Exponentiate it

```
linear = X_des @ beta
print(np.exp(linear))
```

```
[125.08 211.28 154.26 289.38]
```

```
model.predict(X)
```

```
array([125.08, 211.28, 154.26, 289.38])
```

Lecture Outline

- Machine learning workflow
- Data science packages
- Linear Regression Recap
- Generalised Linear Models Recap
- **California House Price Prediction (Setup)**
- California House Price Prediction (Regression)

Data science always starts with the data!

> The target variable is the median house value for California districts, expressed in $100,000's. This dataset was derived from the 1990 U.S. census, using one row per census block group. A block group is the smallest geographical unit for which the U.S. Census Bureau publishes sample data (a block group typically has a population of 600 to 3,000 people).

Dall-E's rendition of this dataset.

Source: Scikit-learn documentation.

Columns

- `MedInc` median income in block group
- `HouseAge` median house age in block group
- `AveRooms` average number of rooms per household
- `AveBedrms` average # of bedrooms per household
- `Population` block group population
- `AveOccup` average number of household members
- `Latitude` block group latitude
- `Longitude` block group longitude
- `MedHouseVal` median house value (**target**)

Source: Scikit-learn documentation.

Import the data

```
features, target = fetch_california_housing(as_frame=True, return_X_y=True)
features
```

	MedInc	HouseAge	AveRooms	AveBedrms	Population	AveOccup	Latitude	Longitude
0	8.3252	41.0	6.984127	1.023810	322.0	2.555556	37.88	-122.23
1	8.3014	21.0	6.238137	0.971880	2401.0	2.109842	37.86	-122.22
2	7.2574	52.0	8.288136	1.073446	496.0	2.802260	37.85	-122.24
...	...	...	...	...	...	...	...	...
20637	1.7000	17.0	5.205543	1.120092	1007.0	2.325635	39.43	-121.22
20638	1.8672	18.0	5.329513	1.171920	741.0	2.123209	39.43	-121.32
20639	2.3886	16.0	5.254717	1.162264	1387.0	2.616981	39.37	-121.24

20640 rows × 8 columns

What is the target?

```
1 target
```

```
0         4.526
1         3.585
2         3.521
          ...
20637     0.923
20638     0.847
20639     0.894
Name: MedHouseVal, Length: 20640, dtype:
float64
```

Why predict this? Let's pretend we are these guys.

The Silicon Valley Elite Who Want to Build a City From Scratch

A mysterious company has spent $800 million in an effort to buy thousands of acres of San Francisco Bay Area land. The people behind the deals are said to be a who's who of the tech industry.

Share full article 1.8K

From left, Michael Moritz, Reid Hoffman, Marc Andreessen and Chris Dixon, four prominent Silicon Valley investors, have backed Flannery Associates. Bloomberg; The New York Times; Clara Mokri for The New York Times; Getty Images; Reuters

Source: Dougherty and Griffith (2023), The Silicon Valley Elite Who Want to Build a City From Scratch, New York Times.

Apply the three-way split to California housing

The same `train_test_split` pattern, now applied to the loaded data:

```
X_main, X_test, y_main, y_test = train_test_split(features, target, test_size=0.2, random
X_train, X_val, y_train, y_val = train_test_split(X_main, y_main, test_size=0.25, random_
print(X_train.shape, X_val.shape, X_test.shape)
```

```
(12384, 8) (4128, 8) (4128, 8)
```

The training set

```
1 X_train
```

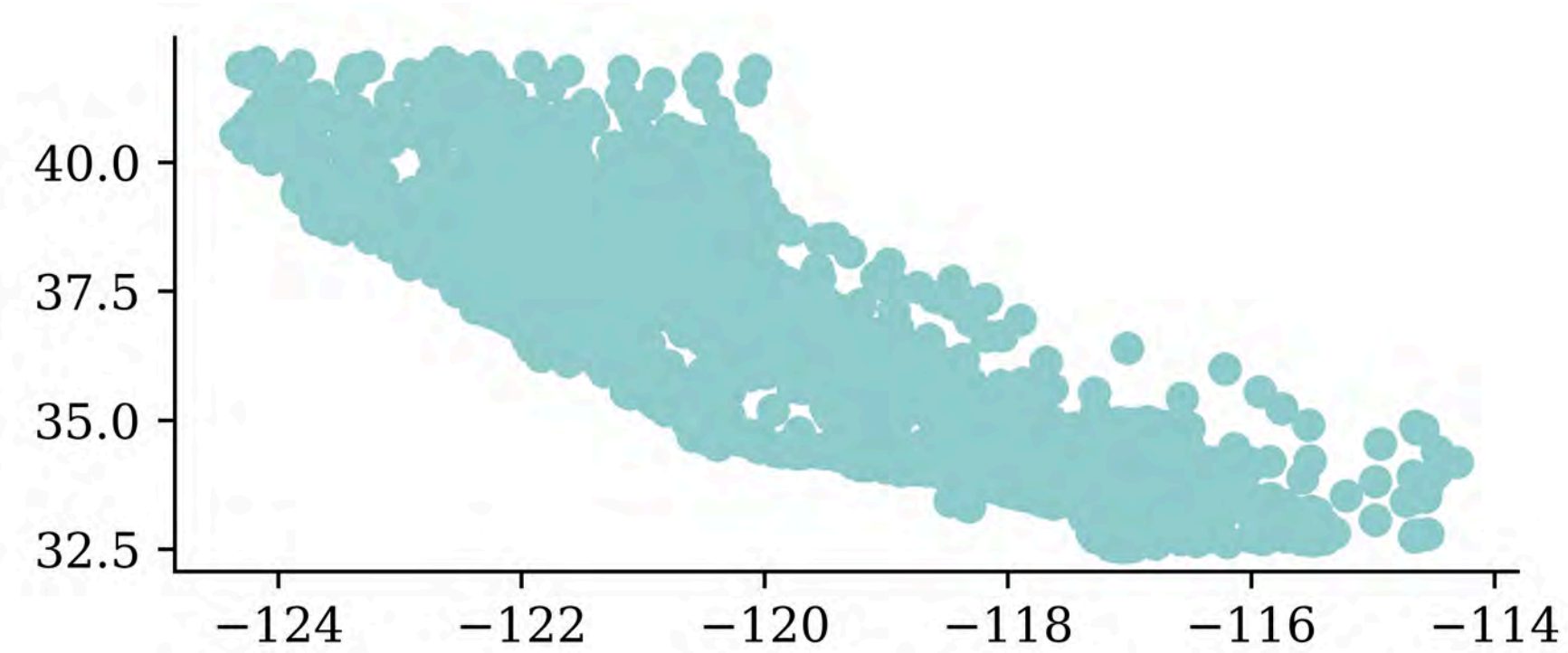
	MedInc	HouseAge	AveRooms	AveBedrms	Population	AveOccup	Latitude	Longitude
9107	4.1573	19.0	6.162630	1.048443	1677.0	2.901384	34.63	-118.18
13999	0.4999	10.0	6.740000	2.040000	108.0	2.160000	34.69	-116.90
5610	2.0458	27.0	3.619048	1.062771	1723.0	3.729437	33.78	-118.26
...	...	...	...	...	...	...	...	...
8539	4.0727	18.0	3.957845	1.079625	2276.0	2.665105	33.90	-118.36
2155	2.3190	41.0	5.366265	1.113253	1129.0	2.720482	36.78	-119.79
13351	5.5632	9.0	7.241087	0.996604	2280.0	3.870968	34.02	-117.62

12384 rows × 8 columns

Location

Python's `matplotlib` package ≈ R's basic `plots`.

```
1 plt.scatter(features["Longitude"], features["Latitude"])
```

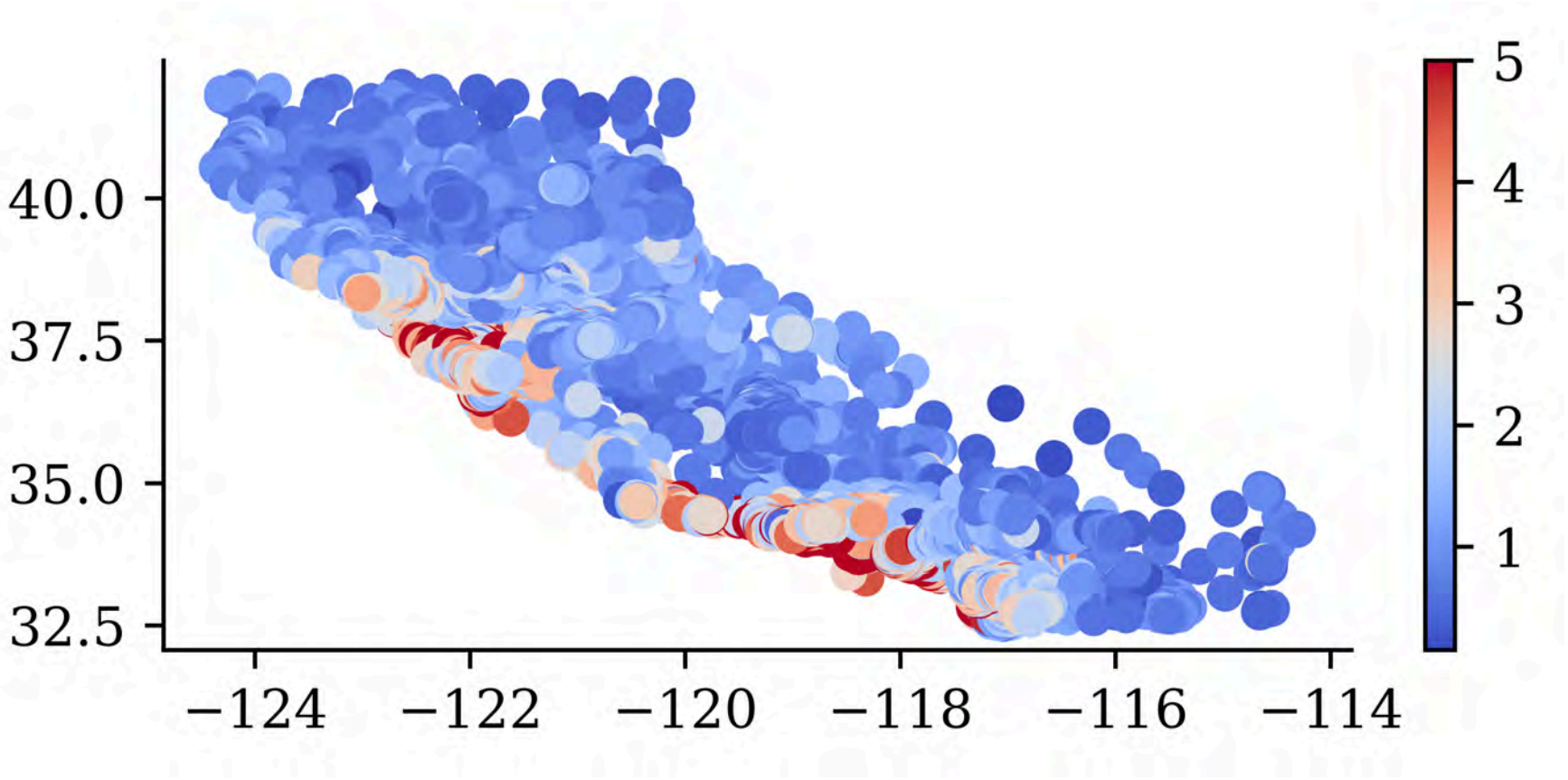


Note

There's no *analysis* in this EDA.

Location EDA

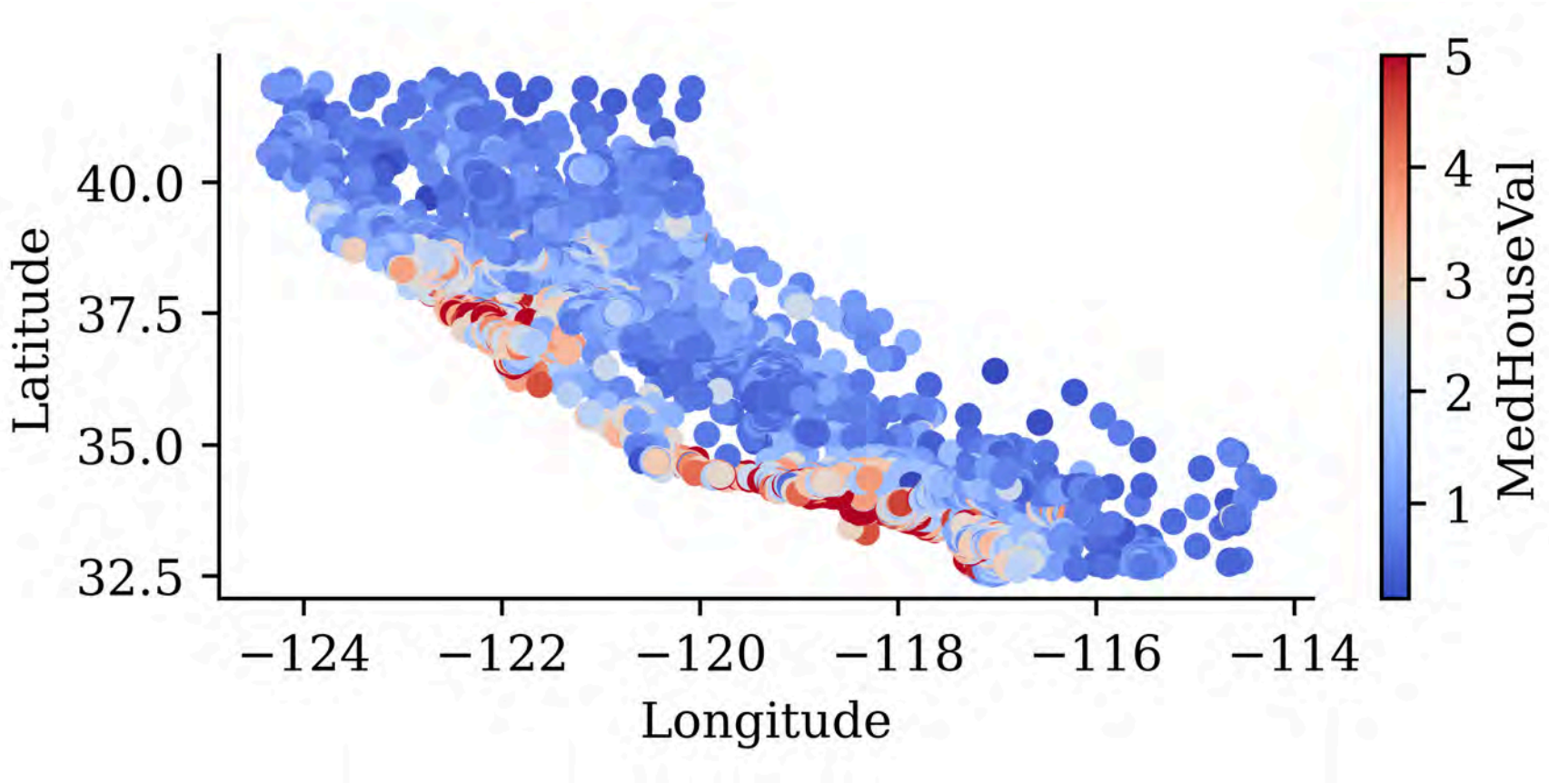
```
1 plt.scatter(features["Longitude"], features["Latitude"], c=target, cmap="coolwarm")
2 plt.colorbar()
```



"We observe that the median house prices are higher closer to the coastline."

Pandas can make plots directly

```
both = pd.concat([features, target], axis=1)
both.plot(kind="scatter", x="Longitude", y="Latitude", c="MedHouseVal", cmap="coolwarm")
```



Features

```
1 print(list(features.columns))
```

```
['MedInc', 'HouseAge', 'AveRooms', 'AveBedrms', 'Population', 'AveOccup', 'Latitude',
'Longitude']
```

How many?

```
1 num_features = len(features.columns)
2 num_features
```

```
8
```

Or

```
1 num_features = features.shape[1]
2 features.shape
```

```
(20640, 8)
```

Lecture Outline

- Machine learning workflow
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- California House Price Prediction (Setup)
- **California House Price Prediction (Regression)**

Linear Regression

$$\hat{y}_i = w_0 + \sum_{j=1}^{p} w_j x_{ij}.$$

```
lr = LinearRegression()
lr.fit(X_train, y_train);
```

The w_0 is in `lr.intercept_` and the others are in

```
print(lr.coef_)
```

```
[ 4.34e-01  9.88e-03 -9.40e-02  5.86e-01 -1.58e-06 -3.60e-03 -4.26e-01
 -4.42e-01]
```

Make some predictions

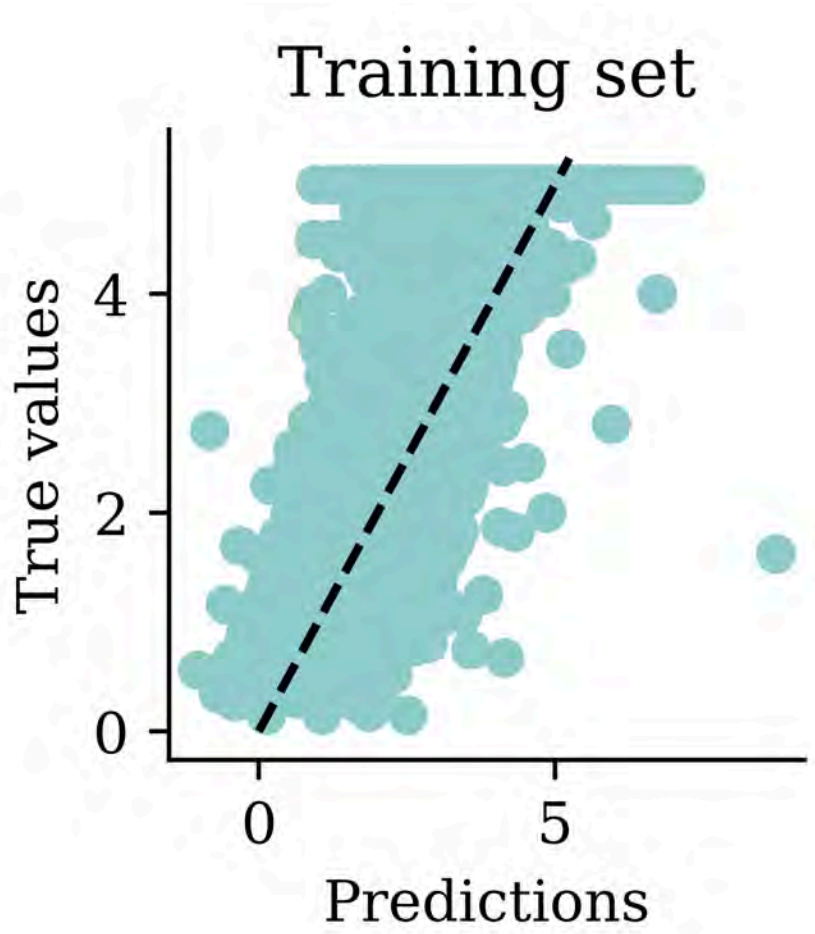
```
1 X_train.head(3)
```

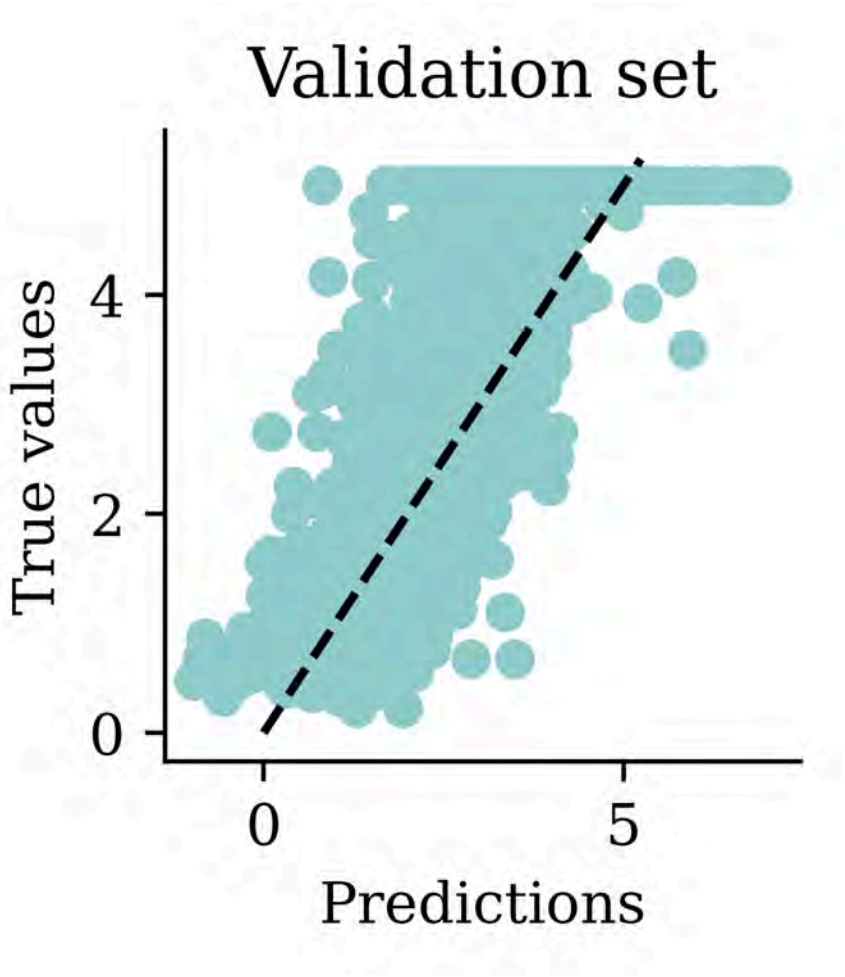
	MedInc	HouseAge	AveRooms	AveBedrms	Population	AveOccup	Latit
9107	4.1573	19.0	6.162630	1.048443	1677.0	2.901384	34.63
13999	0.4999	10.0	6.740000	2.040000	108.0	2.160000	34.69
5610	2.0458	27.0	3.619048	1.062771	1723.0	3.729437	33.78

```
1 y_pred = lr.predict(X_train.head(3))
2 y_pred
```

```
array([1.82, 0.08, 1.62])
```

Plot the predictions





Track MSE for model comparison

We'll add ANN results to these dictionaries later.

```
mse_lr_train = mean_squared_error(y_train, lr.predict(X_train))
mse_lr_val = mean_squared_error(y_val, lr.predict(X_val))

mse_train = {"Linear Regression": mse_lr_train}
mse_val = {"Linear Regression": mse_lr_val}
```

Tip

Think about the units of the mean squared error. Is there a variation which is more interpretable?

To be continued...

We'll revisit this California housing problem in the **AI and Deep Learning** lecture, where we extend `mse_train` and `mse_val` with neural network results and compare them against this linear regression baseline.

Package Versions

```
1 from watermark import watermark
2 print(watermark(python=True, packages="matplotlib,numpy,pandas,seaborn,scikit-learn,scipy
```

```
Python implementation: CPython
Python version       : 3.14.3
IPython version      : 9.13.0

matplotlib  : 3.10.9
numpy       : 2.4.4
pandas      : 3.0.2
seaborn     : 0.13.2
scikit-learn: 1.8.0
scipy       : 1.17.1
```

Glossary

- base case (in dummy encoding)
- categorical input / dummy encoding
- design matrix
- exploratory data analysis (EDA)
- features, target
- generalised linear model (GLM)
- linear regression
- link function
- logistic regression
- matplotlib
- mean squared error
- numpy
- pandas DataFrame
- Poisson regression
- probit regression
- scikit-learn (`fit` / `predict` / `score`)
- sigmoid function
- standardisation (z-score)
- `train_test_split`
- training / validation / test split

References

James, G., Witten, D., Hastie, T., & Tibshirani, R. (2021). *An Introduction to Statistical Learning: with Applications in R*. Springer.