

Rare event estimation: Quiz 1

Email answers & code to Patrick Laub before class on March 14

Consider solving $\ell = \mathbb{P}(M > 3)$ where $M = \max\{X_1, X_2\}$ for $X_i \sim \text{Normal}(0, 1)$ with $\rho = \text{Cor}(X_1, X_2) = 0.8$.

1. Solve (using algebra) the related problem $\tilde{\ell} = \mathbb{P}(\tilde{M} > 3)$ where $\tilde{M} = \max\{\tilde{X}_1, \tilde{X}_2\}$ for $\tilde{X}_i \stackrel{\text{i.i.d.}}{\sim} \text{Normal}(0, 1)$. Would you expect $\tilde{\ell} < \ell$ or $\tilde{\ell} > \ell$? Why?
2. Approximate ℓ using any two different numerical integration algorithms. Which of the two estimates that you created would you expect to be more accurate, and why?
3. Approximate ℓ using crude Monte Carlo with $R = 10^6$. Also give the 99% confidence intervals for the estimate.
4. Approximate ℓ using importance sampling where the X_i 's are no longer standard normals but are $\text{Normal}(\mu, 1)$ (keeping the same correlation). Give the results for a grid of μ values between 0 and 5.25, and recreate the mean/variance plots from the lecture.